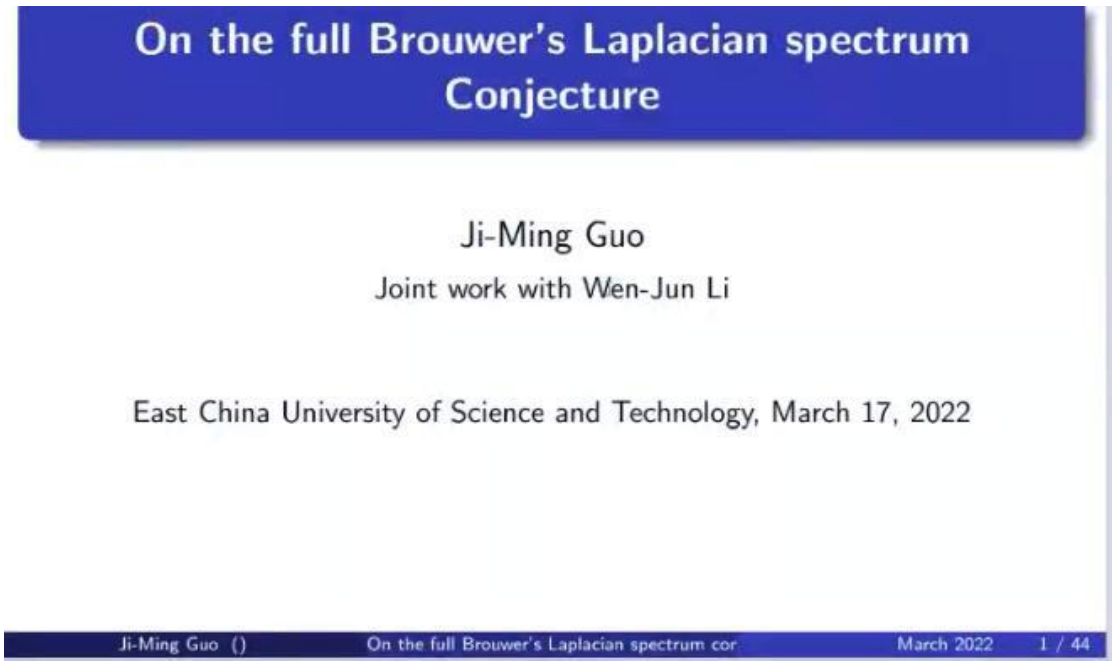
华东理工大学郭继明教授在我院做了学术报告

应我院邀请，华东理工大学数学学院郭继明教授于 2022 年 3 月 17 日北京时间 11 点到 12 点通过腾讯会议对我院师生作了一场学术报告。我院青年教师、研究生等共 40 多人听了报告。

郭继明教授的报告的题目是《On the full Brouwer's Laplacian spectrum conjecture》，该报告主要是关于前 k 大拉普拉斯特征值之和方面的研究工作。在本次讲座中郭继明教授从四个方面进行报告，首先介绍了相关的基础知识和背景以及 Brouwer 的拉普拉斯谱猜想，其次给出了关于 Brouwer 的拉普拉斯谱猜想的一些进展，然后引进一个新图，与 Brouwer 猜想结合，给出了完善的 Brouwer 猜想，即刻画了等号成立的极图，并且证明该猜想对一些特殊的图成立，最后基于图的度序列提出了关于拉普拉斯谱的新猜想并做了尝试。报告后，几位青年教师问了几个问题，郭继明教授作了很详细的解答，并且大家进行了激烈的讨论与交流。



Brouwer's Laplacian spectrum conjecture

A variation on the Grone-Merris conjecture is the following.
In [Brouwer & Haemers, Spectra of graphs, Springer, New York, 2012], Brouwer(2006) proposed the following conjecture which has come to be known as Brouwer's Laplacian spectrum conjecture.

Conjecture 1

For any graph G with n vertices and for any $k \in \{1, 2, \ldots, n\}$,

$$S_k(G) \leq e(G) + \binom{k+1}{2}. \tag{1}$$

Combining with Conjecture 2, we propose the following full Brouwer's Laplacian spectrum Conjecture.

Conjecture 5

For any connected graph G with n vertices, $e(G)$ edges and for any $k \in \{1, 2, \ldots, n-1\}$,

$$S_k(G) \leq e(G) + \binom{k+1}{2},$$

with equality if and only if $G \cong G_{k,r,s}$ $(r \geq 1, s \geq 0)$.

Remark: By using computer computations, we check Conjecture 5 holds for all graphs with at most 9 vertices; for $k = 2$, Conjecture 5 holds for unicyclic graphs and bicyclic graphs.

A new conjecture in terms of Laplacian eigenvalues and degree sequence

In the following, we propose a new conjecture on Laplacian eigenvalues relate to degree sequence of a graph.

Conjecture 6

Let G be a graph with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_n$ and $G \neq C_{4k+1}(k \in Z^+)$. Then

$$\sum_{\mu_i \geq 2} (\mu_i - 2)^2 \leq (1 - \frac{1}{d_1}) \sum_{i=1}^{n} d_i(d_i - 1).$$